(1) Show that elementary matrices are invertible. What are the inverses of each type?

(2) Find solution of the following system of equations in three variables using row-reduction.
$$\begin{aligned} 2x + 4y + 6z &= 22 \\ 3x + 8y + 5z &= 27 \\ -x + y + 2z &= 2 \end{aligned}$$

(3) Find solution of the following system of equations in three variables using row-reduction.
$$\begin{aligned} x + 2y - 3z &= -2 \\ 3x - y - 2z &= 1 \\ 2x + 3y - 5z &= -3 \end{aligned}$$

(4) Find solution of the following system of equations in three variables using row-reduction.
$$\begin{aligned} x + y + z &= 1 \\ 3x - y - z &= 4 \\ x + 5y + 5z &= -1 \end{aligned}$$

(5) Give an example of a system of 4 linear equations in 4 unknowns with infinitely many solutions.

(6) Suppose you have k linear equations in n variables, and by writing in matrix form we have $AX = b$. Give a proof or counterexample for each of the following:
(a) If $n = k$ there is always at most one solution.
(b) If $n > k$ we can always solve $AX = b$.
(c) If $n < k$ then for some b there is no solution of $AX = b$.
(d) If $n < k$ the only solution of $AX = 0$ is $X = 0$.

(7) Consider an arbitrary system of linear equations $AX = B$, where A and B are real matrices.
(a) Prove that if the system of equations $AX = B$ has more than one solution then it has infinitely many.
(b) Prove that if there is a solution in the complex numbers then there is also a real solution.