SWMS 2021

Geometry Worksheet III

The purpose of this worksheet is to motivate Carathéodroy's theorem.

Let C be an m-sided convex polytope in $\mathbb{R}^2$ with vertices $w_1, ..., w_m$.

(a) Describe the set

$$C_2 = \{v \in C : v \text{ is a convex combination of two vertices of } C\}$$

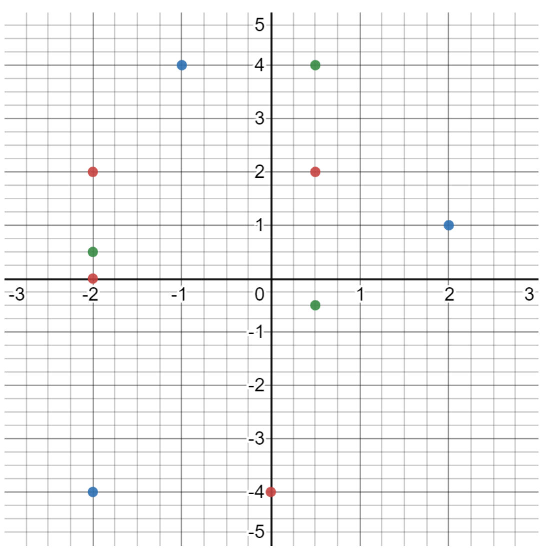
(b) Describe the set

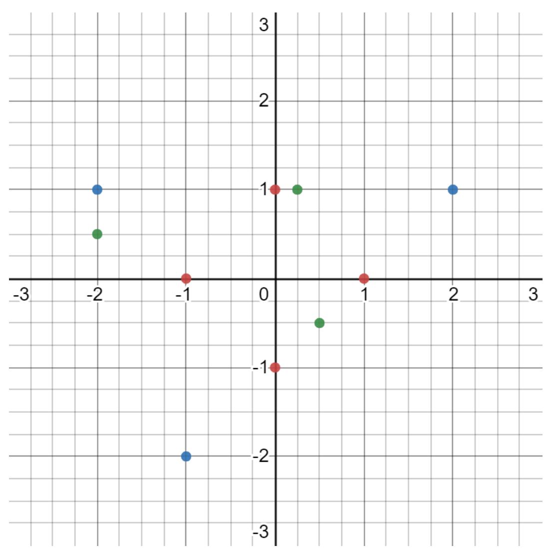
$$C_3 = \{v \in C : v \text{ is a convex combination of three vertices of } C\}$$

(c) Now, let C be the 3-hypercube in $\mathbb{R}^3$. Repeat Parts (a) and (b) for this choice of C. A. Suggest a definition for C_4 and describe C_4 when C is the 3 hypercube. Take other polytopes in $\mathbb{R}^3$, and repeat this exercise.

(d) In each of the following pictures, a monochromatic polytope is a polytope formed by taking the convex hull of all the points of the same color. In each picture, there are three monochromatic polytopes and the origin O is in all them (check this!).

- In each case, can you draw a "rainbow" triangle containing O, i.e., a triangle whose vertices are of different colors? If yes, how mant such rainbow triangles can you find?





- Can you construct a configuration, where this does not happen? I.e., O is in all three monochromatic polytopes, but not in any rainbow triangle?