1. This problem gives one way to find a rational number that approximates π.

   (a) Show that $$\int_0^4 \frac{dy}{1+(y^2/16)} = \pi.$$

   (b) Evaluate the integral using the below R-code that implements the composite Simpson's rule

```
>   options(digits= 22)
>   simp <- function(f, a, b, m = 100) {
+     x.ends = seq(a, b, length.out = m + 1)
+     y.ends = f(x.ends)
+     x.mids = (x.ends[2:(m + 1)] - x.ends[1:m]) / 2 +
+         x.ends[1:m]
+     y.mids = f(x.mids)
+
+     p.area = sum(y.ends[2:(m+1)] + 4 * y.mids[1:m] +
+                     y.ends[1:m])
+     p.area = p.area * abs(b - a) / (6 * m)
+     return(p.area)
+ }
> f <- function(x) { 1/(1+(x^2)/16) }
> simp(f, 0, 4, m = 10)
> simp(f, 0, 4, m = 100)
> simp(f, 0, 4, m = 1000)
> pi
```

   Comment on the three answers and how close they are to π.

   (c) Evaluate the integral using the below R-code that implements the composite Trapezoid rule

```
>    ##Trapezoid
>     options (digits= 22)
>   trap <- function(f, a, b, m = 100) {
+     x = seq(a, b, length.out = m + 1)
+     y = f(x)
+     p.area = sum((y[2:(m+1)] + y[1:m]))
+     p.area = p.area * abs(b - a) / (2 * m)
+     return(p.area)
+ }
> f <- function(x) { 1/(1+(x^2)/16) }
> trap(f, 0, 4, m = 10)
> trap(f, 0, 4, m = 100)
> trap(f, 0, 4, m = 1000)
> pi
```

   Comment on the three answers and how close they are to π.

2. Consider the square of square of size 100 with a corner at the origin. Generate 6 points at random in the square: How would you construct the Convex Hull and how would you compute its area ?

   (a) Using the R-code (Convex hull and Area), that uses the chull and Polygon@area functions answer the above question.

```
> ## Convex Hull
>  # Create 3 sets of random data to plot convex hull around
> x1 <- runif(6, 0, 100)
> y1 <- runif(6, 0, 100)
> X=  cbind(x1,y1)
> plot(X, cex = 0.5)
>       hpts <- chull(X)
>       hpts <- c(hpts, hpts[1])
>       lines(X[hpts, ])
> chull.coords <- X[hpts,]
> chull.poly <- Polygon(chull.coords, hole= F)
> chull.area <- chull.poly@area
```

(b) Rerun the above R-code 3 times and in each trial: Comment on points forming the convex hull; and Comment on the differences in area.

(c) Rerun the above R-code for 5, 7, 8 points.

3. Suppose we have a function $f : [0, 3] \to \mathbb{R}$ given by $f(x) = x\sin(4x)$. Using R-code to decide how well does the optimise function in R find maximum and minimum of functions in the program below.

```
>    ##Optimise
> ## optimize() examples
> f = function(x) x*sin(4*x)
> par(mar= c(4,4,1.5,1.5),mex= .8,mgp= c(2,.5,0),tcl= 0.3)
> curve(f,0,3)
> optimize(f,c(0,3))
> optimize(f,c(1.5,3))
> optimize(f,c(1,3),maximum= TRUE)
> optimize(function(x) -f(x),c(1,3))
```

4. Using the command install.packages("plot3D") install the package in R-Studio. We will now plot the function $f : [-20, 20] \times [-20, 20] \to \mathbb{R}$ given by $f(x, y) = x^2 + y^2$ using the program below

```
> ##Plot3d
> x <- seq(-20, 20, length = 100)
> y <- seq(-20,20, length = 100)
> f = function(x,y){x^2+y^2}
> z = outer(x,y,f)
> require(plot3D)
> persp3D(x,y,z)
> persp3D(x,y,z, theta = 90)
> persp3D(x,y,z, theta = 60)
```

Change the above R to plot the following functions

(a) $f(x, y) = x^2 - y^2$

(b) $f(x, y) = 3x + 7y$

5. Below is an R-code, available here, that solves for Inverses, and Eigen Values of a matrix.

```
> ## LA
> ## Inverse of a matrix
> set.seed(333)
> M = matrix(runif(9), nrow= 3)
```

```
> M
> Minv = solve(M)
> Minv
> Minv%*%M
> zapsmall(Minv%*%M)
> ###############
> ## Eigenvalues and eigenfunctions
> M = matrix(c(2,-1,0,-1,2,-1,0,-1,2), nrow= 3, byrow= TRUE)
> eigen(M)
> #############
```

(a) Describe what the program is doing and its output.

(b) Change the above code and find the inverse of the following matrix using row reduction:

$$A = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}.$$

(c) Change the above and find the eigenvalues of the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & 0 & 3 \end{bmatrix}.$$