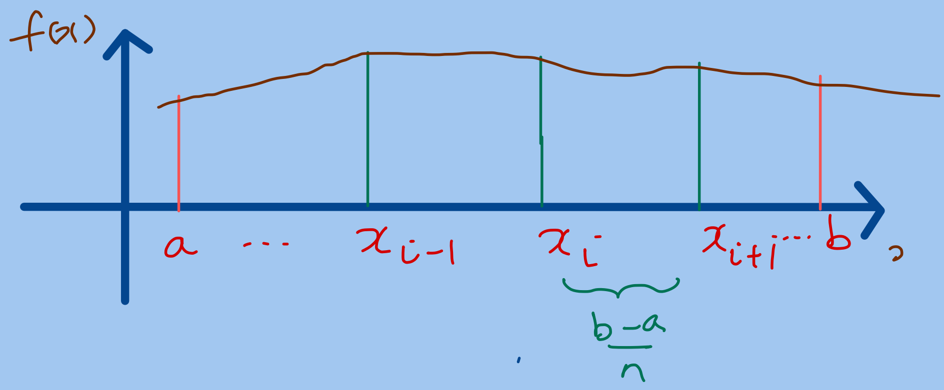
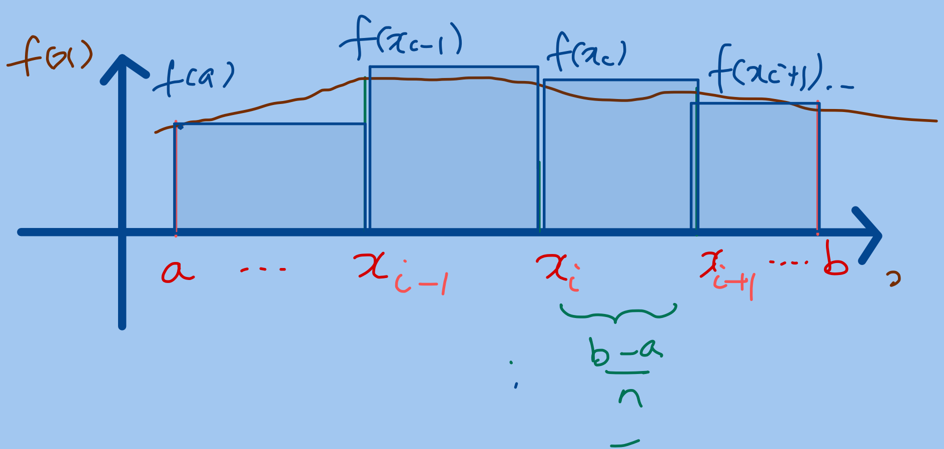
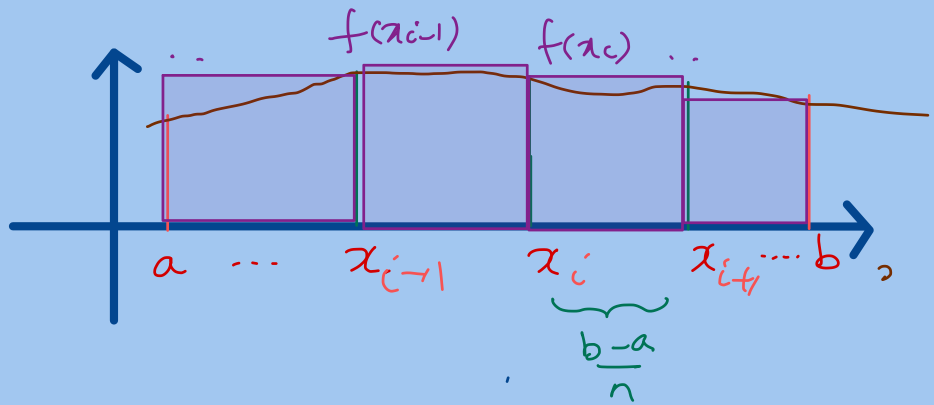


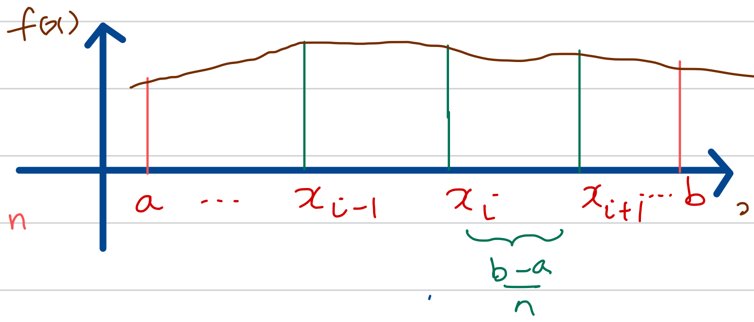




June 16^{th}, 2021

- Riemann Sum.
- Composite Simpsons
- Logic - negation

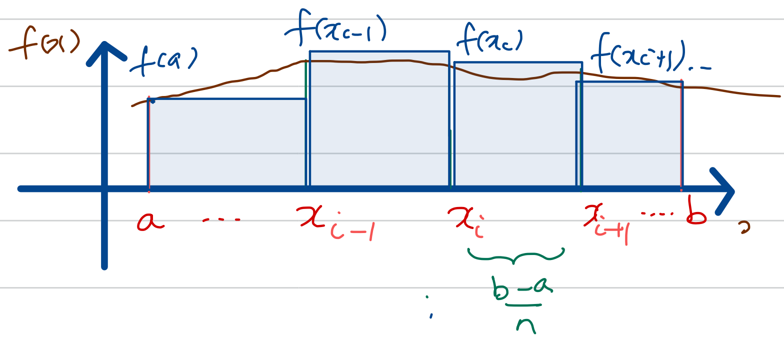
$f : [a,b] \to \mathbb{R}$



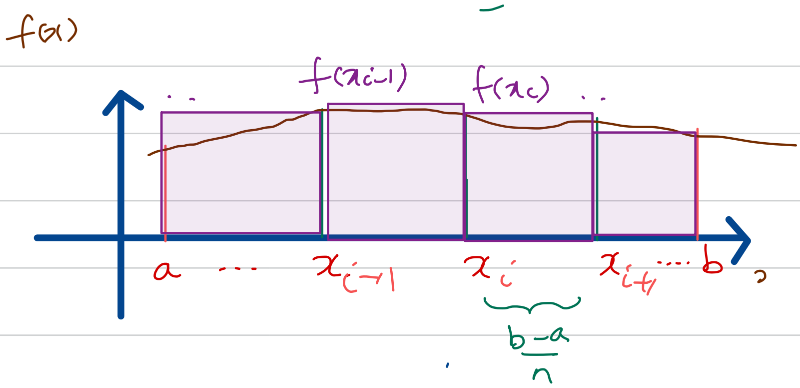
$x_i = \frac{b-a}{n} i + a \qquad i = 0, 1, \dots, n$

Riemann Sums

$$\boxed{L_n = \sum_{k=1}^{n} f(x_{k-1}) \cdot \left(\frac{b-a}{n}\right)}$$



$$\boxed{R_n = \sum_{k=1}^{n} f(x_k) \cdot \left(\frac{b-a}{n}\right)}$$



$$I = \int_a^b f(x)\,dx$$

Question :-

$$|I - R_n| := \alpha_n$$

$$|I - L_n| := \beta_n$$

$$\left|I - \left(\frac{L_n + R_n}{2}\right)\right| := \gamma_n$$

Q Do $\alpha_n, \beta_n, \gamma_n \to 0$ as $n \to \infty$?

Q: How fast do they go to zero?

June 15th, 2021 **Name** ______________________

SWMS- Worksheet 2 in Integral Approximation

1. Suppose we are given the below data about $f : [0,1] \to \mathbb{R}$.

$f(0)$	$f(\frac{1}{2})$	$f(1)$
0	$\frac{1}{\sqrt{2}}$	1

   (a) Use the part (c) to provide an approximation of $\int_0^1 f(x)dx$.

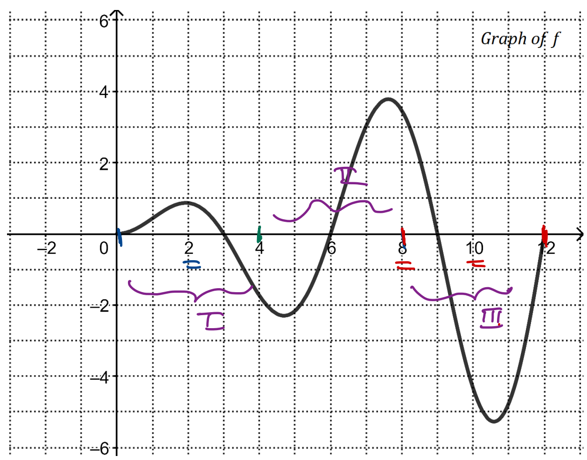
   (b) Suppose $f(x) = \sqrt{x}$ then quantify the error in each approximation.

   (c) Are their functions for which the approximation(s) will be exact ?

2. The graph of a function $f(t)$ is shown. Use it to answer the following questions.

$\frac{1}{8}\int_0^8 f(x)\,dx$ — I

$\frac{1}{8}\int_4^{12} f(x)\,dx$ — II



   (a) Using 1(c) provide an approximation of the **average value** of this function over the interval $[0,8]$. $[4,12]$

   (b) Can you provide a better approximation of the same using 1(c) ?

$\mathrm{I} = \int_0^4 f(t)\,dt \cong [0,4] \equiv \frac{4}{6}[f(0) + 4f(2) + f(4)]$

$\mathrm{II} = \int_4^8 f(t)\,dt \cong [4,8] \equiv \frac{4}{6}[f(4) + 4f(6) + f(8)]$

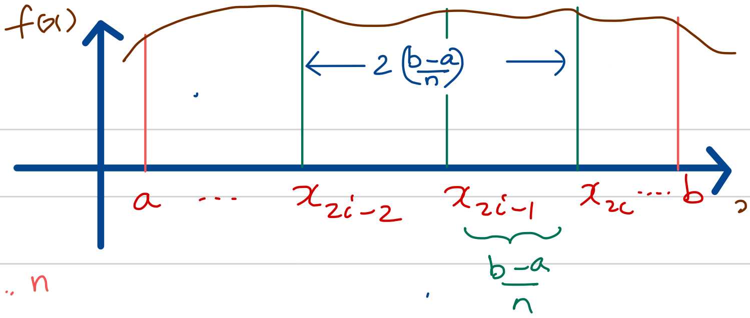
$\mathrm{III} = \int_8^{12} f(t)\,dt \cong [8,12] \equiv \frac{4}{6}[f(8) + 4f(10) + f(12)]$

$\int_0^{12} f(x)\,dx = \mathrm{I} + \mathrm{II} + \mathrm{III}$

$= \frac{4}{6}[f(0) + 4f(2) + 2f(4) + 4f(6) + 2f(8) + 4f(10) + f(12)]$

Ex: Is this better?

$f : [a,b] \to \mathbb{R}$



Simpson's Rule

$x_i = \frac{b-a}{n} i + a \qquad i = 0, 1, .. n$

- Divide into n-even parts

$$I = \int_a^b f(x)\,dx = \sum_{j=1}^{n/2} \int_{x_{2j-2}}^{x_{2j}} f(x)\,dx$$

- Simpsons rule in each part of length $\frac{b-a}{2n}$

$$\Bigg\| \text{Simpson's rule}$$

$$= \sum_{j=1}^{n/2} \frac{2(b-a)}{6n} \left[f(x_{2j-2}) + 4 f(x_{2j-1}) + f(x_{2j}) \right]$$

Algebra

(Ex) $=$

$$\underbrace{\frac{b-a}{3n} \left[f(a) + 2 \sum_{j=1}^{n/2} f(x_{2j-2}) + 4 \sum_{j=1}^{n/2} f(x_{2j-1}) + f(b) \right]}_{(\text{Simpson})_n}$$

Question :- n - even $[a,b]$ into n-parts

$$I = \int_a^b f(x)\,dx$$

$$\alpha_n := I - (\text{Simpson})_n$$

$x_i = \frac{b-a}{n} i + b$

- Does $\alpha_n \to 0$ as $n \to \infty$?
- How fast does it go to 0?

TODAY – OBJECTIVE

Given a sequence $\{a_n\}_{n=1}^{\infty}$

What does

- $a_n \longrightarrow 0$ as $n \to \infty$
- a_n converges to 0 as $n \to \infty$
- $\lim_{n\to\infty} a_n = 0$

?

II

Negation of statement A: a statement B whose assertion specifically denies the truth of statement A.

1. Negate the below statements and express the negations in English avoiding the use of negation words whenever possible.

   (a) There is a vaccine in the world that is not safe for cockroaches. — all vaccine are safe for cockroaches

   (b) All pairs of students who participated in Linear Algebra Mela stood atleast 6 feet apart.

   (c) In Breakout room 1 online classes in SWMS all students were speaking.

   (d) During the month of May, Siva Sanitized his hands every hour.

   there is one hour .... Did not sanitize ...

   (e) There is one person in Immunity's birthday party who is not wearing a mask.

   (f) Every student in this class has taken Bhojpuri or Maithili in Class XII.

   (g) Every student in this class has taken Mathematics and Biology in Class XII.

   There is a student ... not taken ... B and M ...

MAVP

ARSH

ASGMPU

2. Let us introduce **Logical Notation:**
   - $\forall$ to mean for all;
   - $\exists$ to mean there exists;
   - $\implies$ to mean implies; and
   - $\iff$ to mean equivalent.

   Here is an example of usage of notation:
   Statement : For all $\epsilon > 0$ there is an N such that for all $n \geq N$, $a_n \in (a - \epsilon, a + \epsilon)$.
   Statement in logical Notation: $\forall \epsilon > 0$, $\exists N$ such that $\forall n \geq N$, $a_n \in (a - \epsilon, a + \epsilon)$.

   (a) We say $\lim_{n\to\infty} a_n = 5$ if

   For every $\epsilon > 0$ there exists $N > 0$ such that $| a_n - 5 | < \epsilon$ whenever $n \geq N$.

   i. Provide three examples of sequences that converge to 5. $a_n = 5 \ \forall n \geq n$

   ii. Provide three examples of sequences that do not converge to 5.

   iii. Provide an example of sequence that does not converges to any real number.

   iv. Write a logical statement that is equivalent to saying $\lim_{n\to\infty} a_n \neq 5$

   v. Write a logical statement that is equivalent to saying that the sequence a_n does not converge to any real number.

There is one pair of students ......

...... who stood less than 6 feet part

(i) • $a_n = 5 \quad \forall n \geq 1$

• $\{a_n\}_{n\geq 1} = 4.9, \ 4.99, \ 4.999$

$5 - \frac{1}{10}, \ 5 - \frac{1}{100}, \ 5 - \frac{1}{1000}, \ \ldots$

$a_n = 5 - \left(\frac{1}{10}\right)^n$

Ex: $a_n \to 5$ as $n \to \infty$

(ii) $a_n = n$

$a_n = 7$

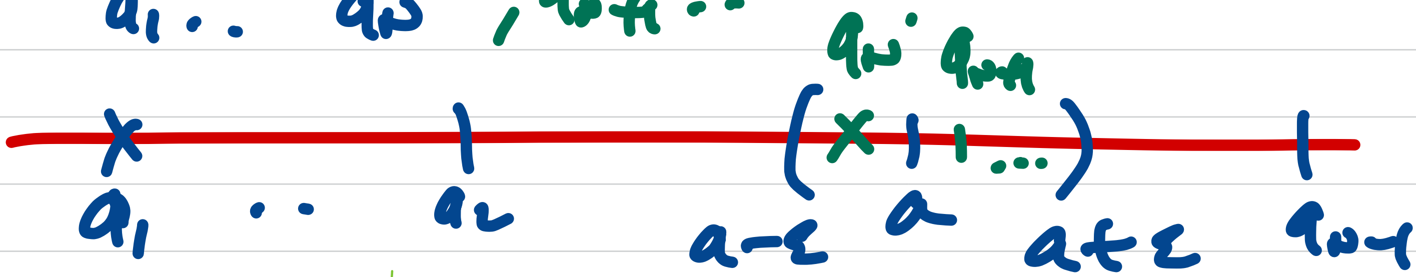
Ex ?

(iii) $a_n = n$

Ex ?

$\forall \varepsilon > 0 \quad \exists N \geq 1$

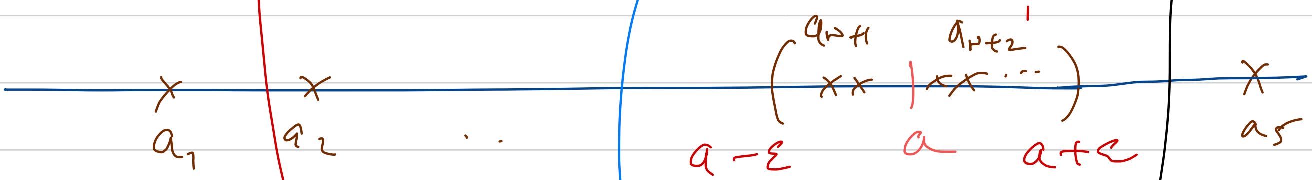
$a_1 \ldots a_N, \ a_{N+1} \ldots$



$\{3,5\}$

$a_n \to a'''$ as $n \to \infty$

$\underline{\forall \varepsilon > 0}$ $\quad \exists N \geq 1$ $\quad$ st $\quad \forall n \geq N$ $\quad \underbrace{|a_n - a|}$ $< \varepsilon$



$\exists \varepsilon_0 > 0 \quad \forall N \geq 1 \quad \exists n_0 \geq N$ st

$|a_n - a| \geq \varepsilon_0$

an does not converge to a

$\underline{\underline{N}}$ $\quad n > N$

Negation of statement A: a statement B whose assertion specifically denies the truth of statement A.

1. Negate the below statements and express the negations in English avoiding the use of negation words whenever possible.
   (a) There is a vaccine in the world that is not safe for lizard.

   Every vaccine in the world is safe for lizard

   (b) All pairs of students who participated in Geometry Mela stood atleast 6 feet apart.

   NSA
   (c) In Breakout room 2 of online class in SWMS all students were silent.
   (d) During the month of May Siva, Sanitized his hands every hour.

   ..... Siva .. there is one hour where he did not Sanitize hand

   FAP
   (e) There is one person in Immunity's birthday party who is not wearing a mask.
   (f) Every student in this class has taken Tulu or Kokborok in Class XII.

   There is one ... ..
   not taken
   T and K ...

   SAH
   (g) Every student in this class has taken Mathematics and Biology in Class XII.
   (h) In every batch of SWMS there is a student who has taken neither Mathematics nor Biology in high school.

2. Let us introduce **Logical Notation:**
   - $\forall$ to mean for all;
   - $\exists$ to mean there exists;
   - $\implies$ to mean implies; and
   - $\iff$ to mean equivalent.

   Here is an example of usage of notation:
   Statement : For all $\epsilon > 0$ there is an N such that for all $n \geq N$, $a_n \in (a-\epsilon, a+\epsilon)$.
   Statement in logical Notation: $\forall\, \epsilon > 0$, $\exists\, N$ such that $\forall n \geq N$, $a_n \in (a-\epsilon, a+\epsilon)$.

   (a) We say $\lim_{n\to\infty} a_n = 3$ if

   For every $\epsilon > 0$ there exists $N > 0$ such that $\mid a_n - 3 \mid < \epsilon$ whenever $n \geq N$.

   i. Provide three examples of sequences that converge to 3.
   ii. Provide three examples of sequences that do not converge to 3.
   iii. Provide an example of sequence that does not converges to any real number.
   iv. Write a logical statement that is equivalent to saying $\lim_{n\to\infty} a_n \neq 3$
   v. Write a logical statement that is equivalent to saying that the sequence a_n does not converge to any real number.

There is one pair of students -----
less than 6 feet apart
- . .