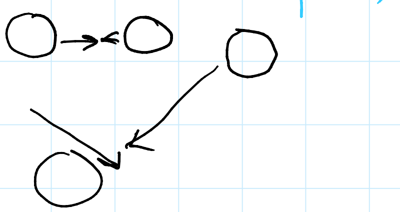
June 24, 2021

SWMS - DISCRETE GEOMETRY

Animation in video 1

Particle

1. Points were floating around inside a box. (speed + direction)
2. Collided (elastic) → other particles (colored) / → frame/walls of the box — " (collision detection) " — coordinates of circles
3. changed direction → direction of collision / → mass of particles / → speed of particles — (collision response)
4. color colliding



(QA) Examples

- gas molecules inside a jar
- billiard balls (dynamics) / carrom
- mob behavior in the event of a crisis
- car racing games → bumping cars / collision detection systems on auto vehicles
- computer games in general
- meteorites
- air bubbles / soap bubbles

many more!

Collision detection (Two objects)

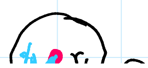
1. Both objects are particles

• (x_1, y_1)

$A = (x_1, y_1)$ $\qquad$ $B = (x_2, y_2)$

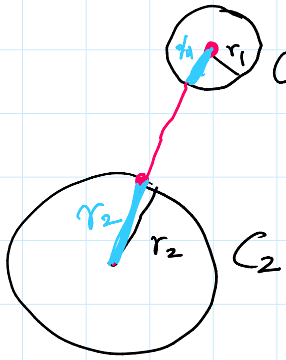
• (x_2, y_2)

$A = B$ iff $d^2(A,B) = (x_1 - x_2)^2 + (y_1 - y_2)^2 = 0.$



$A = B$ iff $d(A,B) = (x_1 - x_2) + (y_1 - y_2) = 0.$

2. Both objects are circles



C_1 : c_1 center (x_1, y_1), radius : r_1

C_2 : c_2 center (x_2, y_2), radius : r_2

C_1 and C_2 do not touch as long as

$$\underbrace{d(c_1,c_2) - (r_1+r_2)}_{\text{distance between 2 circles.}} > 0$$

3. Both objects are (convex) polygons.

Let A and B be two sets in $\mathbb{R}^2$

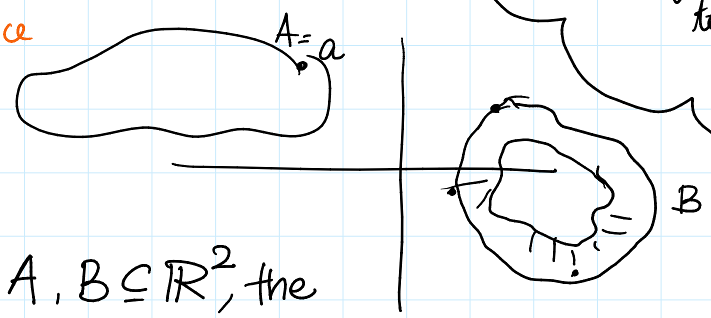
Definition

$$\text{dist}(A,B) = \min\left\{ \underset{=\|a-b\|}{d(a,b)} : \begin{array}{l} a \in A \\ b \in B \end{array} \right\}$$

Muskaan :

Q. If A and B are polygons, can we just take median / some statistic of pairwise distance of vertices?

- Break up into convex bodies.
  ↳ pairwise collision
- "fit" convex bodies to this shape.



Definition Given $A, B \subseteq \mathbb{R}^2$, the Minkowski sum of A & B is the set

$$A + B = \{ a + b : a \in A, b \in B \}.$$

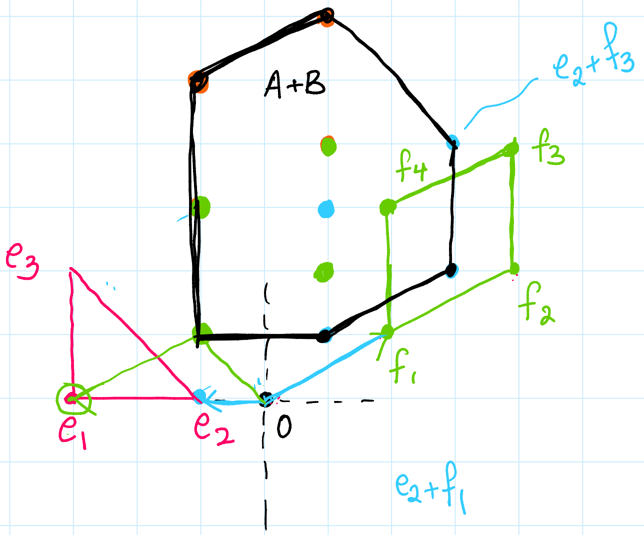
Classroom A

Strategy for adding two polygons
↳ proof
↳ compute some other sums

Classroom B

Find the sum of the following two polygons.

steps: 12 pairings



$$A+B = \{ a+b : a \in A,\ b \in B \}.$$

1. The edges of A+B are always parallel to the edges of either A or B.
2. Perimeter (A+B) = Perimeter (A) + Perimeter (B).

Theorem / Strategy for A+B: In $\mathbb{R}^n$;

$$\operatorname{cvx}(v_1, \dots, v_k) + \operatorname{cvx}(w_1, \dots, w_m) = \operatorname{cvx}\left(v_i + w_j : \begin{matrix} 1 \le i \le k \\ 1 \le j \le m \end{matrix} \right).$$

"complexity": $k \cdot m$.

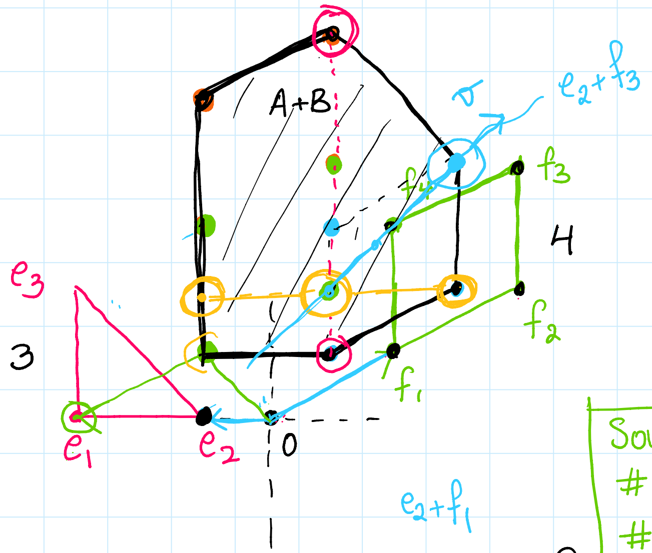
Kanupriya's idea:

Kanupriya's idea:

$\boxed{\text{Take all points maximize } x \,\&\, y \text{ coordinates}}$

Yenisi: Theorem

$$(x+A)+(B) = x+\underline{A+B}$$



Expected # sides of A+B = 12

Actual # of sides of A+B = 6 $\leq$ 3+4

Soumi's Conjecture: # sides of A+B $\leq$ # sides of A + # sides of B

Other operations?

- scalar multiplication: $A \subseteq \mathbb{R}^n$, then $kA = \{ka : a \in A\}$, $k \in \mathbb{R}$.
- subtraction: Minkowski difference

$$(*) \quad A-B = \{a-b : a \in A, b \in B\}$$

$$= A + (-1 \cdot B)$$

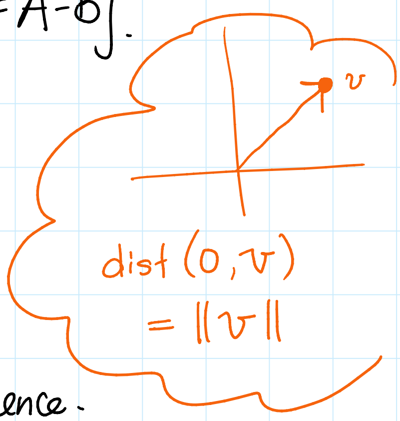
Ex: Compute A−B above
current strategy: 12 steps.

Recall: $(*)$ $\text{dist}(A,B) = \min \{ \|\underbrace{a-b}_{v}\| : a \in A, b \in B\}$. $v \in A-B$

$\|a-b\| = \text{dist}(a,b)$

Q. Can you relate dist (A,B) & A−B?

$$(**) \quad \text{dist}(A,B) = \min \{\|v\| : v \in A-B\}.$$



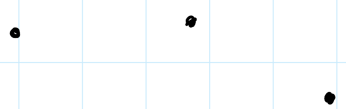
Q. When do A & B collide?

$\iff 0 \in A-B.$

1. Collision detection can be reduced to detecting the origin in the Minkowski difference.

+

2



When can we say that the $0 \in \text{cvx}\{v_1, \dots, v_k\}$?

K?

$v \in \operatorname{cvx}\{v_1, \ldots, v_k\}$:

0 . K ?

NEXT TIME!